Ma3 Task Sheet (L3-6) **Teacher's Guide**

Square Rectangles

This investigation allows pupils to explore formula for finding areas and perimeters of rectangles and squares. This may begin initially with using straightforward measurement or simple addition, and can be extended to identifying and using formulae. There are also opportunities for the use of ICT to quickly identify patterns, and apply formulae.

Less able pupils will probably focus on the initial task, drawing out accurate squares, and measuring areas and perimeters using taught methods, such as counting squares, or measuring with a ruler. They should find, fairly quickly, that the relevant square is that of 4 x 4 units.

Middle range pupils should quickly find the relevant square, and can show that this is unique by drawing a graph to compare increasing perimeter to increasing area. They may also derive a formula to describe the relationships and relate this to the methods for calculating area/perimeter.

More able pupils should resolve the original problem relying largely on data rather than drawings. They may complete graphs/formulae to describe the relationships. They should then move on to consider alternative variations on the theme.

Suggested variations:

- **Find a rectangle which is twice as high as it is wide, that has equal value area & perimeter - one solution**
- **Explore right-angled triangles to find one with area equal to length of two adjacent sides (i.e. excluding hypotenuse) – 2 solutions**
- **Explore the largest possible area achievable from a rectangle of any given perimeter**
- **Gifted pupils may extend this to identify largest possible volume from a cuboid of any given surface area.**

This investigation is limited in its range of achievement to the central 3-6 band, since level 2 pupils are not expected to be able to calculate area/perimeter

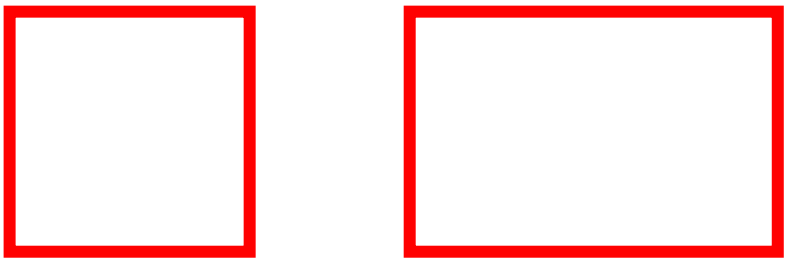
Ma3 Task Sheet (L3-6)

Square Rectangles

Focus: Shape, Space & Measures

In this investigation you will be looking at 2-dimensional shapes. The aim is to explore the relationship between the area and perimeter of shapes – particularly different types of rectangle.

It's useful to remember that a square is just a special type of rectangle which has 4 sides of the same length. You might want to talk with your class to discuss the main features of different shapes – particularly quadrilaterals.



Your Task:

- Try to find a square which has the same measurement around its perimeter as it does for its area (remembering that area is measured in cm^2)
- Draw the shape you've found, and record any other shapes you tried
- Explore other options – can you find a rectangle with one length twice as long as the other which also has perimeter equal to area?

Things to think about:

- Can you show how you calculated these measurements?
- What other investigations could you try? What other shapes?
- How can you record your observations?
- Is there any easy way of finding solutions?

Remember to use the level ladder to help you to achieve your target level.

Key words

rectangle, square, perimeter, area

table, length, width

Ma3 Task Sheet (L3-6)

Square Rectangles

Now use the Level Ladder to achieve your target.

To get level	You should:
3	Accurately draw out some squares to investigate Measure the perimeter of squares With help, count squares to find the area of squares
4	Use your own methods for finding perimeters and areas of squares Record your results in a suitable way, checking for errors Use a method to find an answer to the original problem, noting any patterns
5	Record findings from some initial sketches in a suitable form Use calculations (using ICT as appropriate) to explore relationships Draw some conclusions, using diagrams or graphs
6	Use a calculation method to solve the initial problem, i.e. without drawing all shapes Find a solution to the original problem, explaining whether it is unique – using graphs or diagrams as appropriate Explore further variations of the investigation, based on other shapes or relationships

Now use the Level Ladder to assess your work and decide on improvement targets.

Ma4 Task Sheet (L2-7) **Teacher's Guide**

Pocket Money

This investigation is intended to allow children to demonstrate their understanding of the full cycle of data handling investigations. It would be best preceded with a reminder of the processes involved from setting a question to providing an answer.

The task itself requires pupils to devise their own question based on the topic of pocket money. It would be useful for teachers to guide pupils on the basis of ability to the most appropriate task to allow them to demonstrate their abilities. Of course, more able pupils may start with a simpler task before moving onto something more complex. If they then need to collect further data they can begin to discuss the benefits of broad surveys in data collection.

Less able pupils may focus on tasks such as who earns the most money in a class group, or how much money is given in total for their class.

Middle range pupils can investigate tasks which require the calculation of averages, such as finding the average pocket money earned in a particular group

More able pupils (L5+) should investigate questions which require a comparison between two sets of data, for example comparing whether boys earn more or less pocket money than girls, or whether older pupils earn significantly more than younger ones.

Gifted pupils aiming at high L6, into L7 should incorporate grouped data into their investigation, and be supported to identify limitations on their research, such as small sample sizes, or narrow sample groups.

It will often be useful for teachers to remind pupils of an agreed data handling cycle as they work through their investigations.

The level ladder focuses mainly on the skills required for Ma4, although many of these overlap with the 'Using and applying' requirements of Ma1 by nature.

Ma4 Task Sheet (L2-7)

Pocket Money

Focus: Handling Data

In this investigation you will design your own investigation which will require you to collect some data, and find out some information about it to answer a question. The questions you ask are up to you, but you need to make sure that it is a question you can answer by collecting some data.

Your focus for the investigation should be on pocket money.

You need to decide on your question, carry out some research, and then record and present your data and your findings in the form of a report.

Your Task:

- Decide on a question that you could ask in relation to the main topic
- Select a way of collecting and recording information
- Investigate the data you collect to find some answers to questions
- Present your findings neatly and *clearly*

Things to think about:

- What questions could you ask?
- What data would you need to collect? What might be useful?
- How can you present your data clearly?
- What can you do with the data you have collected?

Remember to use the level ladder to help you to achieve your target level.

Key words

table, graph, average, range, median
mean, mode, maximum, minimum

Ma4 Task Sheet (L2-7)

Pocket Money

Now use the Level Ladder to achieve your target.

To get level	You should:
2	With help, decide what you are going to find out Collect some information from your class Explain what you have found out
3	Devise an appropriate question to ask Collect some data using an appropriate chart or table Present your findings in writing, perhaps using graphs
4	Identify the necessary data to collect to answer your question Collect and present your data in a clear and organised way Calculate the mode and/or range of your collected data
5	Devise a question which requires comparing two sets of data Calculate averages and/or the range of collected data to draw comparisons Present your findings using grouped data and frequency charts
6	Collect data using grouped frequency sets to draw comparisons Present findings using grouped frequency charts, or scattergraphs Explain and justify your findings in response to your original question
7	Devise a question, and make a hypothesis about your study Collect appropriate data in a suitable format Calculate averages of grouped data, for example finding the modal class Identify limitations of your investigation

Now use the Level Ladder to assess your work and decide on improvement targets.

Ma2 Task Sheet (L2-6)

Consecutive Numbers

This extendable investigation commences with very simple calculations involving addition and subtraction, and can be extended to explore relationships between positive and negative numbers and to the use of letters to represent unknown or variable values. It is based on a problem submitted by Bernard Bagnall to the Cambridge enrich website - www.nrich.maths.org

Pupils commence by exploring a selection of 4 consecutive numbers – normally starting with single-digit values. These can be picked at random, or an agreed class set used to begin investigations. The first aspect of the task is to identify all the possible permutations of calculations achievable keeping the digits in order, and using only + and – signs. (There are 8). Then children explore the answers given by their calculations.

Less able pupils will need support to carry out calculations, particularly where these result in negative values. They may take some time to discover the full range of permutations, but will calculate answers for a range of questions, and make some simple observations, e.g. that all answers are even

Middle range pupils should employ a strategy for finding all the permutations of calculations (e.g. starting with +++, leading to ---) and will calculate accurately, including negative values. They should explore different input values, and be able to make general statements about the outputs, e.g. that one solution is always 0. They may begin to devise a test for this statement.

More able pupils will recognise patterns in their results, and should begin to explore representations of the general case, e.g. by using n, n+1, n+2… to represent the consecutive numbers. They will begin to manipulate the algebra to identify the rules behind some of their generalisations.

Once pupils have exhausted the given problem, they may investigate alternatives, such as placing numbers in reverse order, using consecutive even numbers, or using consecutive negative numbers as the starting point.

Ma2 Task Sheet (L2-6)

Consecutive Numbers

Focus: Positive & Negative Numbers

You probably use consecutive numbers all the time. In fact, you've probably been using them since you first learnt to count 1, 2, 3…

In this investigation you will be investigating the patterns you get when you add and subtract consecutive numbers. To do this, you need to choose a set of numbers first of all, and set them out with space in between, like this:

3 4 5 6

The first step of the investigation is to find as many ways as possible of making addition & subtraction sums using the four consecutive digits, like these:

3 + 4 - 5 - 6

3 + 4 - 5 + 6

Once you've found all the possible combinations, you can explore the answers to the sums you've created. It is probably a good idea to write down everything you notice.

Your Task:

- Select your 4 consecutive numbers.
- Try to find all the possible ways of arranging sums using the + and – signs.
- Write down your observations of your answers.
- Investigate other combinations of consecutive numbers.

Things to think about:

- Do you notice any patterns in the answers you get to your sums?
- Do you notice any patterns when you change the consecutive numbers?
- What other variations of the investigation could you try?
- How can you collect and show your findings?

Remember to use the level ladder to help you to achieve your target level.

Key words

combination, pattern, positive, negative

Ma2 Task Sheet (L2-6)

Consecutive Numbers

Now use the Level Ladder to achieve your target.

To get level	You should:
2	Set out your sums clearly Explain anything you notice about the numbers which result Calculate problems using both addition and subtraction
3	List as many possible combinations of problems as possible Check your data for errors, such as repeated problems Record any patterns you notice
4	Use a strategy to list the possible combinations of problems Present results in a clear way, checking for errors Make at least one general statement
5	Systematically represent all the possible combinations of problems Devise a method to represent any sequence of consecutive numbers Make at least one general statement, and support it with examples
6	Devise a method to represent the results of all possible problems Use a generalisation to explain or justify your findings Devise a further investigation

Now use the Level Ladder to assess your work and decide on improvement targets.

Ma3 Task Sheet (L3-7) **Teacher's Guide**

Polyominoes

This is a common and popular investigation based on 2-dimensional shapes made up of squares. The term polyominoes is best linked to dominoes for pupils' understanding. They should begin to understand that poly- means many, and should recognise the common prefixes for other units, e.g. tri-, pent-, hex-, etc.

The activity can be easily introduced with the use of resources to represent the construction of polyominoes – 2d versions if possible, although multilink or unifix will also work. Pupils can make dominoes together, then explore triominoes. This will allow opportunities for the teacher to explain the rules of constructing polyominoes and recognising that different polyominoes can be formed, but that some are reflections or rotations of existing designs.

Less able pupils will explore tetrominoes, finding all 5 designs, and recognising why other variations are not permitted. They will begin to explore other features of tetrominoes, for example identifying those which have reflective symmetry.

Middle range pupils will explore pentominoes, identifying all 12 designs. They will explore other aspects of the designs, and may begin to identify generalisations, such as having a common area. They may also identify the differences in perimeter and begin to explain these.

More able pupils will work with pentominoes and may look at the 36 hexomino designs. They will explore the symmetries of the shapes, and may begin to look at issues such as tessellation in the plane. They may also explore other variations, such as using 5 multilink cubes to make 3-dimensional shapes.

This investigation can be well supported with the use of physical resources throughout, and can be used to allow pupils to begin to explore 2-dimensional representations of 3D shapes, for example by identifying which pentominoes can be made into an open box, or which hexominoes are nets of cubes.

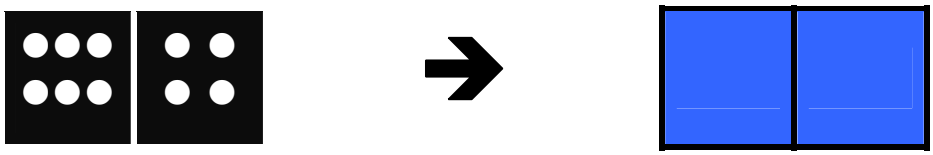
Ma3 Task Sheet (L3-7)

Polyominoes

Focus: Shape, Space & Measures

In this investigation you will begin by looking at combinations of 2-dimensional shapes. Of course, once you've done this, it's up to you to devise your own extensions to the problems to make the mathematics more interesting & challenging for you!

Lets start first of all with something you know: a domino. We can think of a domino as being made up of two squares:



There is only one way in which two squares can be put together to make a shape by joining edges – you will always get a rectangle. You can turn it, or reflect it, but it will still be the same shape: there is only one type of domino.

Now you can explore other combinations of squares to make shapes.

Your Task:

- Start by investigating *tri*ominoes. How many of them can you find?
- What about tetrominoes?
- Are there any methods you can use to find all the possible combinations?
- Extend your investigation yourself in some way.

Things to think about:

- How can you record your findings?
- Is there a limit on the number of shapes?
- Can you transfer your work into three dimensions?
- What else could you investigate about the properties of these shapes?

Remember to use the level ladder to help you to achieve your target level.

Key words

domino, triomino, tetromino, pentomino, hexomino, polyomino

reflection, rotation, symmetry, area, perimeter

Ma3 Task Sheet (L3-7)

Polyominoes

Now use the Level Ladder to achieve your target.

To get level	You should:
3	Draw out all the varieties of triomino Find as many different types of triomino Explain why there are no more triominoes, or which triominoes didn't count
4	Find all the varieties of tetromino Check that you have no duplicated tetrominoes, explaining where you do Present your results clearly Explore one other aspect of tetrominoes, for example, reflection
5	Use a strategic approach to finding all tetrominoes and pentominoes, explaining why some have to be discounted Identify other features of tetrominoes, or pentominoes, including reflectional and rotational symmetry Explore the relationships between perimeter and area in different tetrominoes or pentominoes
6	Use a strategic approach to find all pentominoes, explaining where some have been discounted using mathematical language Systematically record all varieties of pentomino, and may extend this to hexominoes Explore other features of pentominoes or hexominoes, such as those which can be used to create nets of 3D shapes
7	Extend your investigation beyond the framework set for pentominoes & hexominoes, perhaps by considering 3D arrangements, or using other starting shapes Investigate other aspects of shapes produced, for example surface area, or volume Seek to explore general statements about some aspects of your work

Now use the Level Ladder to assess your work and decide on improvement targets.

Ma4 Task Sheet (L3-7) **Teacher's Guide**

Easy winnings

This investigation allows pupils to demonstrate their understanding of probability as a form of comparison of success. It is based on the idea of comparing the performance of a pair of dice to a spinner. Teachers may choose to introduce the unit by playing with dice & spinners to allow children to get a grasp on the games played, and a sense of the fairness & inequalities involved.

Less able pupils will begin by randomly recording possible combinations from dice throws. They may begin to organise their results, and spot duplications. This should lead to them devising a strategy for recording results. They may then begin to explain why the probability of rolling a double-six is not the same as spinning a 12.

Middle range pupils will record outcomes from dice rolls systematically, perhaps listing in order. They will quickly identify the reason for the inequality, and may be able to describe it using mathematical language in terms of probability. They may find a way to address the problem.

More able pupils should quickly resolve the initial problem, referring to it in mathematical terms, and using a two-way table to record results. They should devise fairer ways of playing the game, referring to equal probabilities, and may suggest designing a new spinner. They should go on to explore other aspects of the game – for example whether the spinner allows one player to move further forward more frequently than the dice.

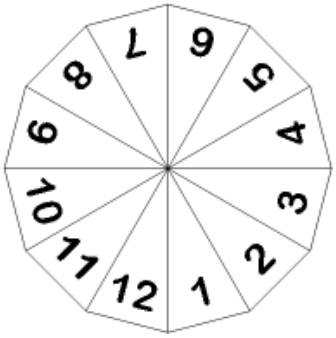
The investigation allows pupils to explore theoretical probability. It also allows an opportunity for the key comparison between theoretical and experimental probability. Pupils could play a simple game in pairs which involves moving around a board, and record how often each type of counting device wins. This could lead to discussions as to whether the theoretical probability is always right, and of the usefulness of experimental probability to model situations.

Ma4 Task Sheet (L3-7)

Easy winnings

Focus: Probability

James and Sasha play a board game online which uses two dice to move around the board. Sasha doesn't have two dice, but uses a 12-sided spinner. The rules say that each player moves along the board the number of spaces shown on the two dice together. For example, throwing a 3 and a 6 means the player can move 9 spaces. An extra go is awarded each time the player rolls a double-six – James and Sasha agree that Sasha earns her extra 'roll' if she spins a '12'.



After playing several games, James claims that Sasha must be cheating because she gets many more extra 'rolls' than him on every game.

Your Task:

- Try to decide whether James is right. Must Sasha be cheating, or is there some other explanation for her luck?
- Use probability to explain why this problem has arisen
- Try to devise a fairer way to play the game.

Things to think about:

- What is the chance that each player will score a bonus point?
- What difference does using the spinner make?
- Are there any other problems which might occur?
- How can you collect and show your findings?

Remember to use the level ladder to help you to achieve your target level.

Key words

bias, equal, event, fair, unfair,
chance, outcome, possible, probability

Ma4 Task Sheet (L3-7)

Easy winnings

Now use the Level Ladder to achieve your target.

To get level	You should:
3	List as many possible outcomes of the dice throws as possible Check your data for errors Describe the number of chances each player has of earning an extra 'roll'
4	List all the possible outcomes of the dice throws Include all variants of the dice throw outcomes Give a reason for Sasha's apparent success
5	Systematically list all the possible outcomes of dice throws Represent probabilities as either a fraction, decimal or percentage Explain a reason for Sasha's apparent success Devise a fairer way of allocating extra 'rolls'
6	Use a two-way table to record all the possible outcomes of dice throws Check probabilities by totalling for all outcomes, and compare the resulting probabilities for the two methods Devise a fairer way of running the game with the existing tools Consider how the spinner could be made more fair
7	Compare the probabilities of getting an extra 'roll' with both methods Examine another aspect of the game, for example, whether both players can expect to achieve the same number of moves over the course of a game Consider how the spinner could be made to 'mimic' the behaviour of the die Conduct an experiment with a mock-up spinner to see if you have created an improved product

Now use the Level Ladder to assess your work and decide on improvement targets.

Ma2 Task Sheet (L2-7) **Teacher's Guide**

Sequences

This investigation allows pupils to develop their understanding of sequences, and to demonstrate their use of algebra to describe and explain a sequence.

Most pupils will be able to start with the pattern given on the introductory sheet, although teachers may choose to cover simpler patterns in earlier lessons. The task given requires pupils to recognise and continue a pattern and then to develop their understanding of the sequences involved.

Less able pupils will continue the pattern by drawing 4th and 5th diagrams, before collecting some simple information, perhaps in a table.

Middle range pupils should record data based on the sequence, and begin to identify patterns. Many will be able to describe these relationships using simple algebra, e.g. T=4d+1 for total number of squares.

More able pupils will create formulae to explain each aspect of the sequence, leading to an overall formula. They should draw the graph of their formula, and will begin to connect the structure of the formula to the form of the graph (y=mx+c). They should also justify their findings referring back to the original diagram.

Pupils can choose to investigate further patterns. Low & middle ability pupils should initially focus on patterns of a similar structure, where the number of blocks increases by a common value (e.g. adding 5 blocks each time). More able pupils should move to look at sequences which lead to quadratic equations, for example:

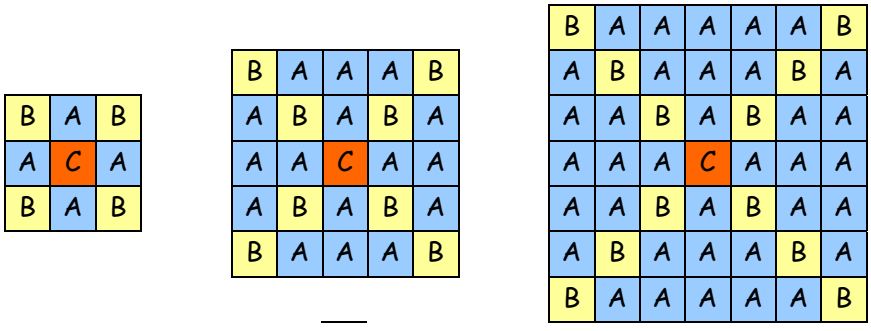


Diagram 1 Diagram 2 Diagram 3

for which the final formula can be derived: $T=4d^2+4d+1$

Pupils working at this level may recognise that the formula for the total area could also be calculated using $T=(2d+1)^2$. Gifted pupils working at level 7 may be able to manipulate the algebra to confirm that the two equations are equal using expansion of brackets, or less commonly, factorisation.

Ma2 Task Sheet (L2-7)

Sequences

Focus: Sequences, Functions & Graphs

In this investigation you will be looking at patterns in sequences that you will construct yourself. You will be given one to start you off, and then some ideas you may choose to pursue if you like.

Remember, with all investigations you should clearly show all your drawings, explanations, graphs & charts for the reader. Firstly, look at this sequence:

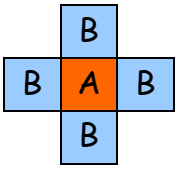


Diagram 1

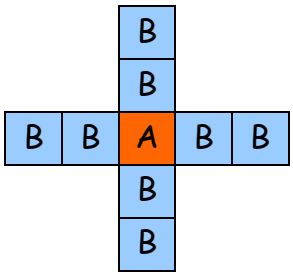


Diagram 2

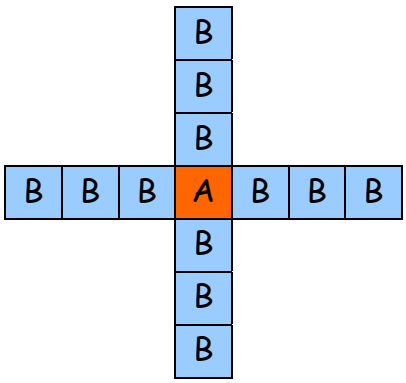


Diagram 3

Your Task:

- You may choose to continue the sequence in your book
- You should try to find a way to record any patterns in the sequence
- You should try to describe any patterns you find in words
- You may be able to describe patterns using algebra

Things to think about:

- How can you record all the details of the sequence?
- How could you represent any patterns you spot?
- Can you find any links, or rules?
- How can you collect and show your findings?

Remember to use the level ladder to help you to achieve your target level.

Key words

sequence, pattern, equation, formula, graph

axis, gradient, y-intercept, position, term

Ma2 Task Sheet (L2-7)

Sequences

Now use the Level Ladder to achieve your target.

To get level	You should:
2	Continue the sequence by drawing the next 2 or 3 diagrams Make at least on statement about any patterns you notice With help from your teacher, use a table to record some information
3	Use a table to record some information about the sequence Describe any patterns you notice in the sequence Test your statements by extending the pattern
4	Use a table to help you to identify patterns Present your results in another form, such as a graph Try to describe in words any patterns you've found in the table
5	Explore different ways of representing your findings Investigate another sequence Write your own statements based on what you have found
6	Investigate a more complex sequence Relate your graph to the formula found, or sequence used Begin to explain why your formula works
7	Investigate a more complex sequence, including square numbers Explain the link between the sequence, the algebra & its graphical form Explain how you know that your formula will always be true

Now use the Level Ladder to assess your work and decide on improvement targets.